Design of the Question Paper

MATHEMATICS (35) **CLASS : II PUC**

Time: 3 hours 15 minute; Max. Mark:100
(*of which 15 minutes for reading the question paper*).
The weightage of the distribution of marks over different dimensions of the question paper shall be as follows:

I. Weightage to Objectives:

Objective	Weightage	Marks
Knowledge	40%	60/150
Understanding	30%	45/150
Application	20%	30/150
Skill	10%	15/150

II. Weightage to level of difficulty:

Level	Weightage	Marks
Easy	35%	53/150
Average	55%	82/150
Difficult	10%	15/150

III. Weightage to content:

Chapter No.	Chapter	No. of teaching Hours	Marks
1	RELATIONS AND FUNCTIONS	11	11
2	INVERSE TRIGONOMETRIC FUNCTIONS	8	8
3	MATRICES	8	9
4	DETERMINANTS	13	12
5	CONTINUITY AND DIFFERENTIABILITY	19	20
6	APPLICATION OF DERIVATIVES	11	10
7	INTEGRALS	21	22
8	APPLICATION OF INTEGRALS	8	8
9	DIFFERENTIAL EQUATIONS	9	10
10	VECTOR ALGEBRA	11	11
11	THREE DIMENSIONAL GEOMETRY	12	11
12	LINEAR PROGRAMMING	7	7
13	PROBABILITY	12	11
	Total	**150**	**150**

IV. Pattern of the question paper:

PART	Type of questions	Number of questions to be set	Number of questions to be answered	Remarks
A	1 mark questions	10	10	Compulsory part
B	2 mark questions	14	10	---
C	3 mark questions	14	10	---
D	5 mark questions	10	6	Questions must be asked from the specific set of topics as mentioned below, under section V
E	10 mark questions (Each question with two subdivisions namely a) 6 mark and b) 4 mark.	2	1	

V. Instructions:

Content areas to select questions for PART – D and PART – E

a) In PART D:

1. Relations and functions: Problems on verification of invertibility of a function and writing its inverse.

For example:

a. Show that the function, $f : R \to R$ defined by $f(x) = 4x + 3$ is invertible. Hence write the inverse of f.

b. Let R_+ be the set of all non-negative real numbers. Show that the function $f : R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$ is invertible. Also write the inverse of $f(x)$.

c. If R_+ is the set of all non-negative real numbers prove that the function $f : R_+ \to [-5, \infty)$ defined by $f(x) = 9x^2 + 6x - 5$ is invertible. Write also $f^{-1}(x)$.

2. Matrices: Problems on verifications of basic conclusions on algebra of matrices.

For example:

a. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -2 \\ 3 & 0 \end{bmatrix}$ verify that $A(BC) = (AB)C$.

b. If $A' = \begin{bmatrix} 1 & 5 \\ 2 & 0 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$ find $A + B$ and $B - C$ show that $A + (B - C) = (A + B) - C$.

c. If $A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix}$ verify that $AB - BA$ is a skew symmetric matrix and $AB + BA$ is a symmetric matrix.

3. Determinants: Problems on finding solution to simultaneous linear equations involving three unknown quantities by matrix method.

For example:

a. Solve the following by using matrix method:
$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 4,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 4.$

b. Solve the following by using matrix method:
$2x + y + z = 1,\ x - 2y - z = \frac{3}{2},\ 3y - 5z = 9.$

c. Use the product $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$ to solve the system of equations $x - y + 2z = 1,\ 2y - 3z = 1,\ 3x - 2y + 4z = 9.$

4. Continuity and differentiability: Problems on second derivatives.

For example:

a. If $x = a(\cos t + t.\sin t)$ and $y = a(\sin t - t.\cos t)$ find $\frac{d^2y}{dx^2}$.

b. If $(x-a)^2 + (y-b)^2 = c^2,\ c > 0,$ prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

c. If $e^y(x+1) = 1$ prove that $\frac{dy}{dx} = -e^y$. Hence prove that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

5. Application of derivatives: Problems on derivative as a rate measurer.

For example:

a. A ladder $5\,m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of $2\,cm/sec$. How fast is its height of the wall decreasing when the foot of the ladder is $4\,m$ away from the wall?

b. Sand is pouring from a pipe at the rate of $12\,cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is $4\,cm$?

c. A stone is dropped into a quit lake and waves move in circles at a speed of $4\,cm\ per\ second$. At the instant, when the radius of the circular wave is $10\,cm$, how fast is the enclosed area increasing?

6. Integrals: Derivations on indefinite integrals and evaluation of an indefinite integral by using the derived formula.

For example:

- **a.** Find the integral of $\frac{1}{\sqrt{x^2 - a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2 - 25}}\, dx$.
- **b.** Find the integral of $\sqrt{a^2 - x^2}$ with respect to x and hence evaluate $\int \sqrt{1 + 4x - x^2}\, dx$.
- **c.** Find the integral of $\frac{1}{x^2 - a^2}$ with respect to x and hence evaluate $\int \frac{x}{x^4 - 16}\, dx$.

7. Application of integrals: Problems on finding the area of the bounded region by the method of integration.

For example:

- **a.** Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.
- **b.** Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.
- **c.** Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle, $x^2 + y^2 = 32$.

8. Differential equations: Problems on solving linear differential equations.

For example:

- **a.** Solve the differential equation, $x\frac{dy}{dx} + 2y = x^2 \log x$.
- **b.** Find a particular solution of the differential equation $\frac{dy}{dx} - 3y.\cot x = \sin 2x$, $y = 2$, when $x = \frac{\pi}{2}$.
- **c.** Find the equation a curve passing through the point $(0, 1)$.If the slope of the tangent to the curve at any point (x, y) is equal to the sum of x coordinate and the product of x coordinate and y coordinate of that point.

9. Three dimensional geometry: Derivations on 3 dimensional geometry (both vector and Cartesian form).

For example:

- **a.** Derive a formula to find the shortest distance between the two skew line $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ in the vector form.
- **b.** Derive the equation of a plane passing through three non collinear points both in the vector and Cartesian form.
- **c.** Derive the equation of a line in space passing through two given points both in the vector and Cartesian form.

10. Probability: Problems on Bernoulli trials and binomial distribution.

For example:

a. A die is thrown 6 times. If "getting an odd number" is a success, what is the probability of (i) 5 success? (ii) at least 5 successes?

b. Five cards are drawn successively with replacement from a well shuffled deck of 52 playing cards. What is the probability that (i) all the five cards are spades? (ii) only 3 cards are spades? (iii) none is a spade?

c. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs (i) none will fuse after 150 days. (ii) at most two will fuse after 150 days.

b) In PART E:

(i) 6 mark questions must be taken from the following content areas only.

1. Integrals: Derivations on definite integrals and evaluation of a definite integral using the derived formula.

For example:

a. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ and hence evaluate $\int_0^{\pi/2} \log(\sin x)\,dx$.

b. Prove that $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ and hence evaluate $\int_{-1}^{2} |x^3 - x|\,dx$.

c. Prove that $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x) & if\ f(2a-x) = f(x) \\ 0 & if\ f(2a-x) = -f(x) \end{cases}$ and hence evaluate $\int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2 \sin^2 x}\,dx$.

2. Linear programming: Problems on linear programming.

For example:

a. A corporative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much should be allocated to each crop so as to maximize the total profit of the society?

b. Solve the following linear programming problem graphically:

Minimize and maximize $Z = x + 2y$, subject to constraints
$x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x,\ y \geq 0$

c. There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

(ii) 4 mark questions must be taken from the following content areas only.

1. Continuity and differentiability: Problems on continuous functions.

For example:

a. Verify whether the function $f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ x^2, & \text{if } x < 0 \end{cases}$ is continuous function or not.

b. Find the points of discontinuity of the function $f(x) = x - [x]$, where $[x]$ indicates the greatest integer not greater than x. Also write the set of values of x, where the function is continuous.

c. Discuss the continuity of the function $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if} -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$.

2. Determinants: Problems on evaluation of determinants by using properties.

For example:

a. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$.

b. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

c. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

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SAMPLE BLUE PRINT

II PUC MATHEMATICS (35)

TIME: 3 hours 15 minute **Max. Mark: 100**

Chapter	CONTENT	Number of Teaching hours	PART A	PART B	PART C	PART D	PART E		Total marks
			1 mark	2 mark	3 mark	5 mark	6 mark	4 mark	
1	RELATIONS AND FUNCTIONS	11	1	1	1	1			11
2	INVERSE TRIGONOMETRIC FUNCTIONS	8	1	2	1				8
3	MATRICES	8	1		1	1			9
4	DETERMINANTS	13	1	1		1		1	12
5	CONTINUITY AND DIFFERENTIABILITY	19	1	2	2	1		1	20
6	APPLICATION OF DAERIVATIVES	11		1	1	1			10
7	INTEGRALS	21	1	2	2	1	1		22
8	APPLICATION OF INTEGRALS	8			1	1			8
9	DIFFERENTIAL EQUATIONS	9		1	1	1			10
10	VECTOR ALGEBRA	11	1	2	2				11
11	THREE DIMENSIONAL GEOMETRY	12	1	1	1	1			11
12	LINEAR PROGRAMMING	7	1				1		7
13	PROBABILITY	12	1	1	1	1			11
	TOTAL	**150**	**10**	**14**	**14**	**10**	**2**	**2**	**150**

GUIDELINES TO THE QUESTION PAPER SETTER

1. The question paper must be prepared based on the individual blue print without changing the weightage of marks fixed for each chapter.

2. The question paper pattern provided should be adhered to.

Part A : 10 compulsory questions each carrying 1 mark;

Part B : 10 questions to be answered out of 14 questions each carrying 2 mark ;

Part C : 10 questions to be answered out of 14 questions each carrying 3 mark;

Part D : 6 questions to be answered out of 10 questions each carrying 5 mark

Part E : 1 question to be answered out of 2 questions each carrying 10 mark with subdivisions (a) and (b) of 6 mark and 4 mark respectively.

(The questions for PART D and PART E should be taken from the content areas as explained under section V in the design of the question paper)

3. There is nothing like a single blue print for all the question papers to be set. The paper setter should prepare a blue print of his own and set the paper accordingly without changing the weightage of marks given for each chapter.

4. Position of the questions from a particular topic is immaterial.

5. In case of the problems, only the problems based on the concepts and exercises discussed in the text book (prescribed by the Department of Pre-university education) can be asked. Concepts and exercises different from text book given in Exemplar text book should not be taken. Question paper must be within the frame work of prescribed text book and should be adhered to weightage to different topics and guidelines.

6. No question should be asked from the historical notes and appendices given in the text book.

7. Supplementary material given in the text book is also a part of the syllabus.

8. Questions should not be split into subdivisions. No provision for internal choice question in any part of the question paper.

9. Questions should be clear, unambiguous and free from grammatical errors. All unwanted data in the questions should be avoided.

10. Instruction to use the graph sheet for the question on LINEAR PROGRAMMING in PART E should be given in the question paper.

11. Repetition of the same concept, law, fact etc., which generate the same answer in different parts of the question paper should be avoided.